P425/1
PURE MATHEMATICS
Paper 1
Nov./Dec. 2020
3 hours

WAKISO-KAMPALA TEACHERS' ASSOCIATION (WAKATA)

WAKATA MOCK EXAMINATIONS 2020

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All necessary working must be clearly shown.

Begin each answer on a fresh sheet of paper.

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

Neat work is a must!!

 Turn Over

SECTION A (40 MARKS)

Answer **all** *questions in this section.*

1. By reducing to Echelon form, solve the simultaneous equations

$$2x - y + z = 5$$
$$x - 3y + 2z = 2$$
$$2x + y + 4z = -3$$

(05marks)

2. Solve the equation $1 - cos^2\theta = -2sin\theta cos\theta$, for $-180^0 \leq \theta \leq 180^0$ *(05marks)*

3. Use first principles to differentiate $\frac{8}{\sqrt{x}}$ with respect to x. *(05marks)*

4. Find the perpendicular distance from the point $P(4i + 2j + 2k)$ to the line $r = 3i + j - k + \lambda(i - j + 2k)$. *(05marks)*

5. Find the real values of k for which the equation $x^2 + (k+1)x + k^2 = 0$ has real roots *(05marks)*

6. The point $A(at^2, 2at)$ lies on the parabola $y^2 = 4ax$ and B is the point $(-a, 2a)$. M is the mid point of AB. Find the Cartesian equation of the locus of M as B moves on the parabola. *(05marks)*

7\. Show that $\int_0^{\frac{\pi}{2}} \frac{4\, d\theta}{3 + 5sin\theta} = In3$ *(05marks)*

8. Solve the differential equation $(x+2)\frac{dy}{dx} = (2x^2 + 4x + 1)(y - 3)$ given that $x = 0$ when $y = 7$ *(05marks)*

SECTION B (60 MARKS)

Answer any **five** *questions from this section. All questions carry equal marks.*

9. (a) If Z_1 and Z_2 are complex numbers, solve the simultaneous equations $4Z_1 + 3Z_2 = 23$, $Z_1 + iZ_2 = 6 + 8i$, giving both answers in the form $x + yi$. *(05marks)*

(b) Show by shading on an Argand diagram the region in which $2 \leq |Z - i| \leq 3$ *(07marks)*

10. (a) Show that the $x -$ coordinates of any points of intersection of the line $y = mx + c$ and the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ are given by the solutions of the quadratic equation $(4 + 9m^2)x^2 + 18mcx + (9c^2 - 36) = 0$ *(04marks)*

(b) If the line $y = mx + c$ is a tangent to the ellipse in (a) above, prove that $c^2 = 4 + 9m^2$. *(04marks)*

(c) The line $y = mx + c$ in (b) above passes through the point $(2,3)$. Write down a second equation connecting m and c, and hence prove that m must satisfy the equation $5m^2 + 12m - 5 = 0$ *(04marks)*

11. (a) Find the equation of the normal to the curve $x^3 + 3x^2y = 2y^2$ at the point $(-1, 1)$ *(06marks)*

(b) Given that $y = x\sqrt{x + 3}$, show that $\frac{dy}{dx} = \frac{3(x+2)}{2\sqrt{(x+3)}}$. *(06marks)*

12. (a) Prove that $cos(45^0 - \alpha)\, cos(45^0 - \beta) - sin(45^0 - \alpha) sin(45^0 - \beta) = sin(\alpha + \beta)$ *(05marks)*

(b) If $A, B,$ and C are the angles of a triangle, prove that $cosA + \cos(B + C) \equiv 0$. Hence deduce that $cosA + \cos(B - C) = 2sinBsinC$ *(07marks)*

13. (a) Express $\frac{6-2x}{(x+1)(x^2+3)}$ in partial fractions.

Hence evaluate $\int_{-1}^{3} \frac{6 - 2x}{(x + 1)(x^2 + 3)} dx$. *(12marks)*

14. (a) Lines L_1, L_2 and L_3 are given by equations $\frac{x+5}{2} = \frac{y-14}{-10} = \frac{z+13}{11} = \lambda$, $\frac{x-3}{2} = \frac{y+5}{-3} = \frac{z+17}{-5} = \mu$ and $x = y + 5 = \frac{z-7}{-2} = \gamma$ respectively.

Given that L_1 intersects L_2 at point A while L_1 intersects L_3 at B, find the distance AB. *(07marks)*

(b) Find the coordinates of the point of intersection of the line $\frac{x+3}{2} = \frac{y-5}{-1} = \frac{z-2}{-3} = \mu$ and the plane $2x + 7y + 5z - 3 = 0$ *(05marks)*

15. (a) Prove by induction that $7^n + 2^{2n+1}$ is always divisible by 3 for all positive integral values of n. *(05marks)*

(b) Given that the first three terms in the expansion in ascending powers of x of $(1 + 2x)^{½}$ are the same as the first three terms in the expansion of $\frac{1+ax}{1+bx}$.

(i) Find the values of the constants $\boldsymbol{a}$ and $\boldsymbol{b}$.

(ii) Hence show that an approximation of $\sqrt{2}$ for $x = -\frac{1}{100}$ is $\frac{1970}{1393}$. *(07marks)*

16. In a certain type chemical reaction a substance A is continuously transformed into a substance B. Throughout the reaction the sum of the masses of A and B remains constant and equal to m. The mass of B present at time, t after the commencement of the reaction is denoted by x. At any instant the rate of increase of the mass of B is k times the mass of A, where k is a positive constant.

(a) Form a differential equation relating x and t, hence solve the equation, given that $x = 0$, when $t = 0$ *(01mark)*

(b) Given also that $x = ½m$ when $t = In2$, determine the value of k, and show that, at time t, $x = m(1 - e^{-t})$ *(07marks)*

(c) Hence find

(i) the value of x (in terms of m) when $t = 3In2$,

(ii) the value of t when $x = ¾m$ *(04marks)*

END